MATH 8 UNIT 3

More Graphs, Identities and Trig Equations

NAME: ____

	Unit 3 HW Checklist	
NOTES	completed and in order given including completd worksheets:	10
	WS page 4 solving trig eqns 2	5
	WS page 10-11 graphing the other trig functions	5
	WS page 17 Solving trig eqns 3	5
11.2i	1, 3, 7, 9, 13, 17, 22, 25, 39, 45, 49, 51	5
11.2ii	74, 76, 77, 89, 91	5
11.2iii	55, 59,67, 68, 69, 78, 110, 111, 116, 117	5
11.4	5, 7, 8, 16, 19, 21, 29, 36, 37, 39, 41, 53-57 <=hint: sum to product	5
11.3i	1-25 odd, 188-191, 194, 198, and page 1015 25, 27, 28, 31	5
B2ii	11, 12, 36, 37, 40	5
	Sample Test	10
		65

Unit 3– Graphs of the Other Trig Functions, Identities, Inverse Trig and Equations.

More Solving Trigonometric Equations (covered in 11.4 of text, I break into pieces and cover differently)

Multistep Equations: First isolate the trig function first, then solve for the argument

1) Solve: $2\cos(\theta)-1=0; \quad 0\le\theta<2\pi$

2) Solve $\tan^2(t)-1=0$

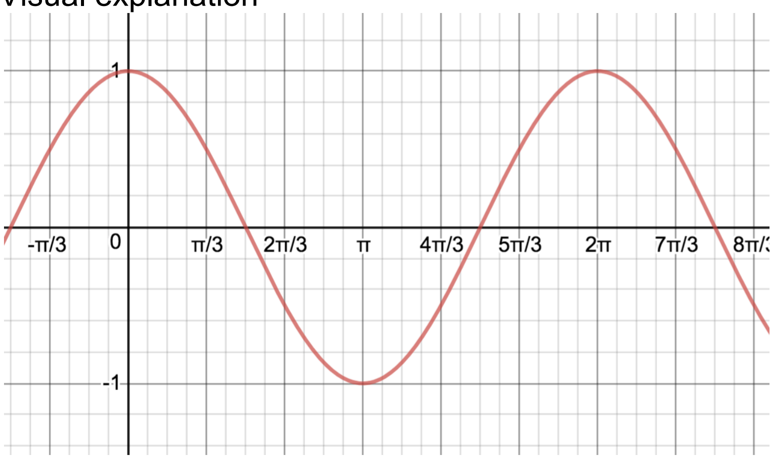
Solving when there is an expression in the argument. First solve for the argument, then the variable.

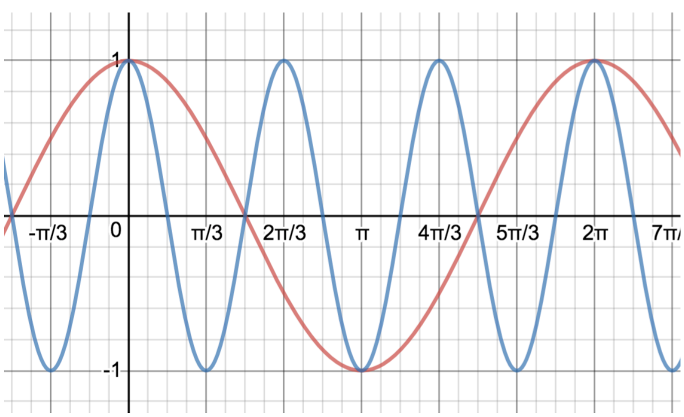
3) Solve: $\sin(x-3)-1=0$

4) Solve $\tan\left(\frac{x}{3}\right)=\sqrt{3}$

5) Solve $\cos(3\theta)+1=0; \quad 0 \le \theta < 2\pi$ *******

Visual explanation





Worksheet – Solving Trigonometric Equations – part ii

(1) Solve the following equations exactly. (all solutions)

(a) $\cos\theta = \frac{\sqrt{2}}{2}$

Answers: $\theta = \frac{\pi}{4} + 2\pi k, \quad \frac{7\pi}{4} + 2\pi k \quad k \in Z$

(b) $\sin\left(\frac{x}{2}\right) = \frac{-\sqrt{3}}{2}$

$x = \frac{8\pi}{3} + 4\pi k, \quad \frac{10\pi}{3} + 4\pi k \quad k \in Z$

(c) $\tan 3\theta = -1$

$\theta = \frac{\pi}{4} + \frac{\pi}{3}k, \quad k \in Z$

---

(2) Solve the following equations exactly for $0 \le \theta \le 2\pi$.

(a) $\sec\theta = -2$

Answers: $\theta = \frac{2\pi}{3}, \ \frac{4\pi}{3}$

(b) $\tan^2\theta = 3$

$\theta = \frac{\pi}{3}, \ \frac{2\pi}{3}, \ \frac{4\pi}{3}, \ \frac{5\pi}{3}$

(c) $\sin(2\theta) = -\frac{1}{2}$

$\theta = \frac{7\pi}{12}, \ \frac{11\pi}{12}, \ \frac{19\pi}{12}, \ \frac{23\pi}{12}$

---

(3) Solve the following equations exactly for $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$.

(a) $2\cos\theta + 1 = 0$

Answers: no solution

(b) $\tan(\theta) + 1 = 0$

$\theta = \frac{-\pi}{4}$

(c) $\sin(4\theta) = \frac{\sqrt{2}}{2}$

have to think on this one

$\theta = \frac{-7\pi}{16}, \ \frac{-5\pi}{16}, \ \frac{\pi}{16}, \ \frac{3\pi}{16}$

Unit 3– Graphs of the Other Trig Functions, Identities, Inverse Trig and Equations.

10.5 Graphs of Tangent, Cotangent, Cosecant and Secant.

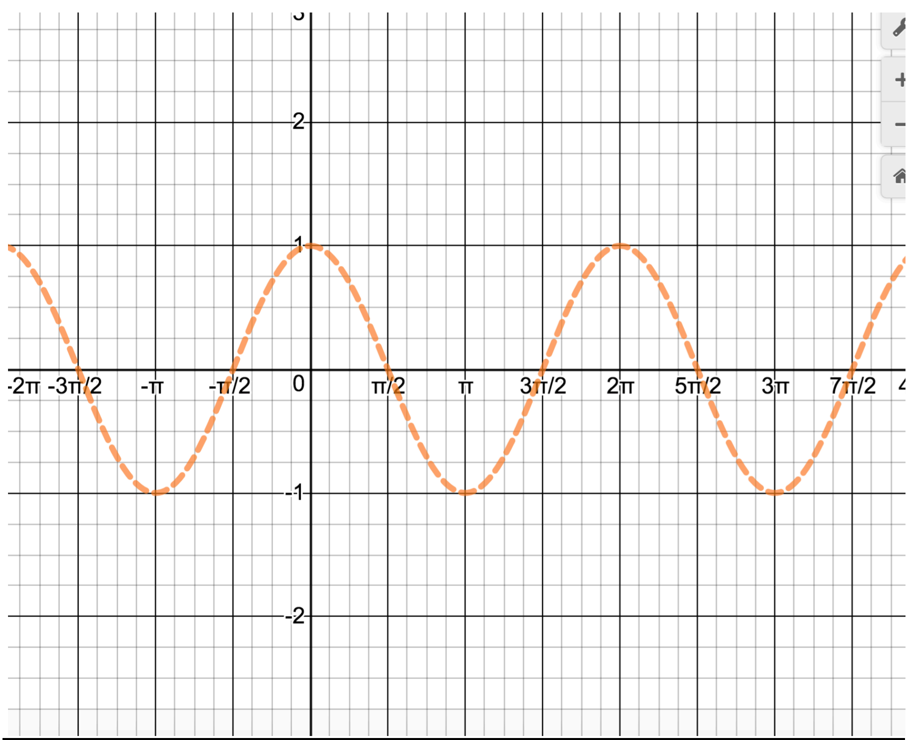
For the graphs of secant and cosecant, we can use our knowledge of graphing cosine and sine, together with the reciprocal relationships:

Ex: $f(x)=\sec(x)$

Since $f(x)=\sec(x)=\dfrac{1}{\cos(x)}$, everywhere that ____________________, $f(x)=\sec(x)$ is ______________, thus where the graph of $g(x)=\cos(x)$ has an ______________________________ the graph of

$f(x)=\sec(x)$ has a _______________________.

Also, as $g(x)=|\cos(x)|$ gets small $f(x)=|\sec(x)|$ _______________

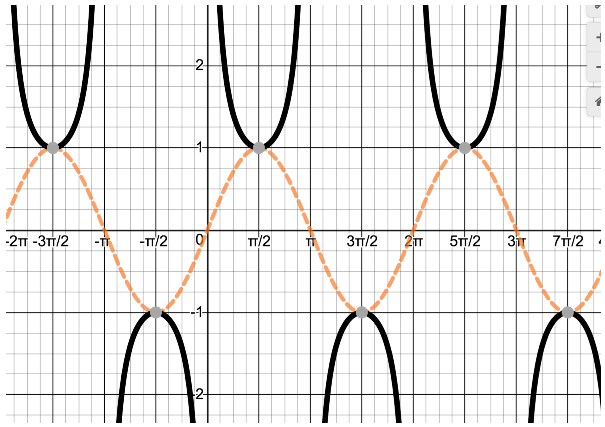


Previously, by the unit circle definition, we determined the domain of $f(x)=\sec(x)$ to be

_________________________________ which we can now see on the graph. In addition, the graph tells us that the range of $f(x)=\sec(x)$ is _____________________________________

What is the period of $f(x)=\sec(x)$? _____________________________________

The graph of $f(x)=\csc(x)$ can be found similarly (see text).



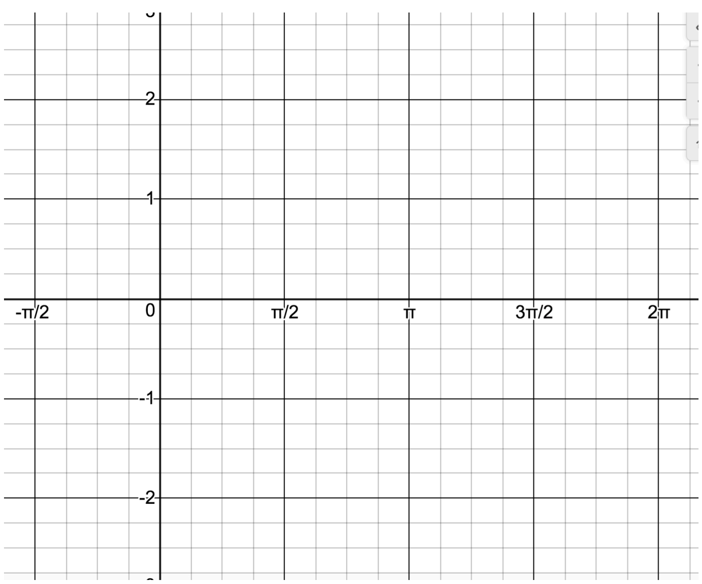
Notice the location of the vertical asymptotes. How would we find them algebraically?
Note domain, range, period

How would you graph $f(x)=\csc\left(x-\dfrac{\pi}{4}\right)$?

Note: Even though we could use the above graph together with a

transformation (______________________________), it is actually easier to

____

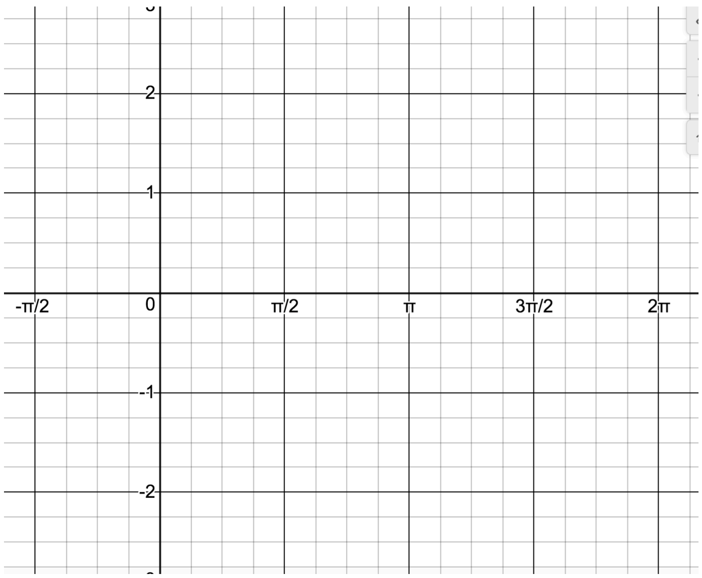


EX: Sketch the graph of $f(x) = 3\csc(2x)$.

Note: Even though we could use the above graph together with a

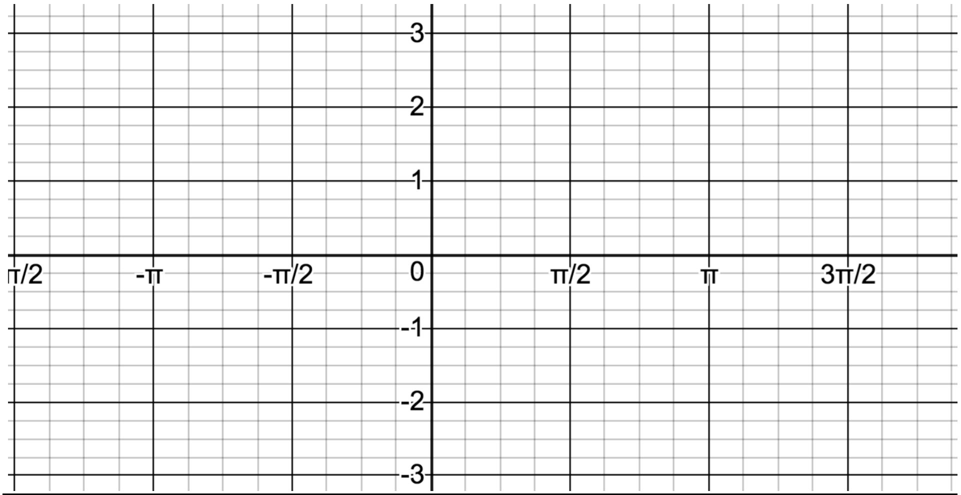
transformation (____________________________), it is actually easier to

____



How would we find the domain and asymptotes algebraically if we didn't have the graph?

The graph of $f(x) = \tan(x)$



Discuss domain, range, period, odd, asymptotes…

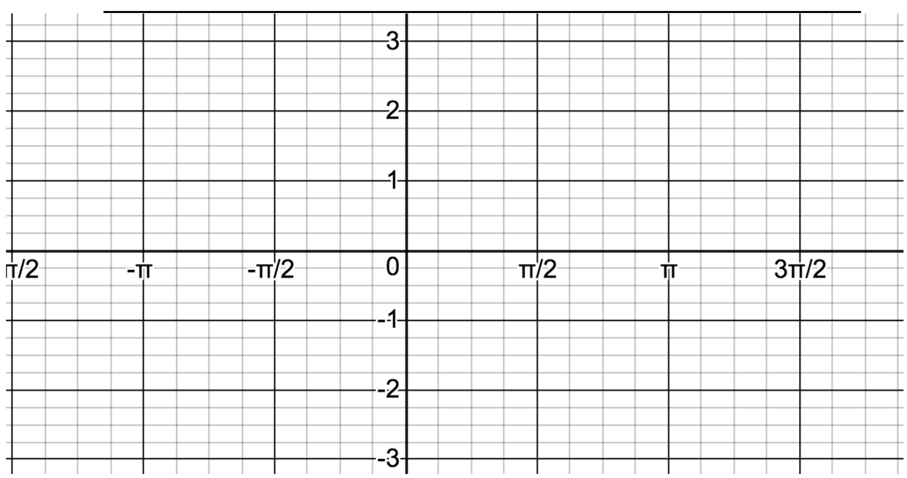
EX: Sketch the graph of $f(x) = \tan(2x)$

.

Note: Even though we could use the above graph together with a

transformation (______________________________), it is actually easier to

____

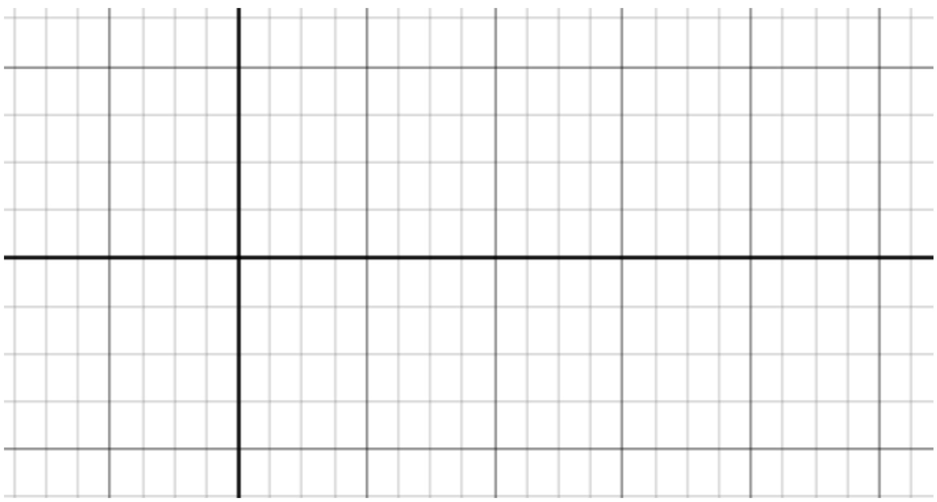


What is the period? ________________________________

In general $f(x) = A\tan(\omega x)$ has period____________________ Find asymptotes by considering where the denominator $\cos(\omega x) = 0$, . Midway between asymptotes is an x intercept, midway between the x intercept and an asymptote, f(x)=A or -A

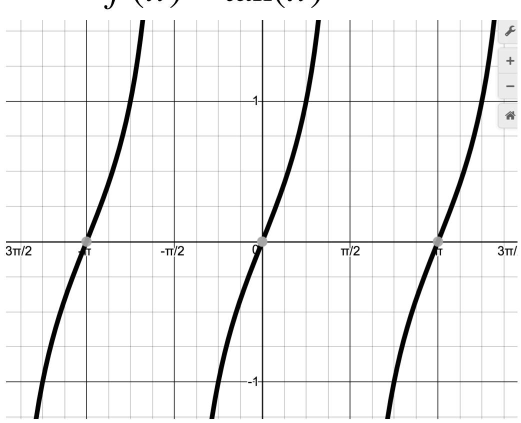
EX: Sketch the graph of $f(x) = 2\tan\left(\dfrac{\pi}{3}x\right)$

.

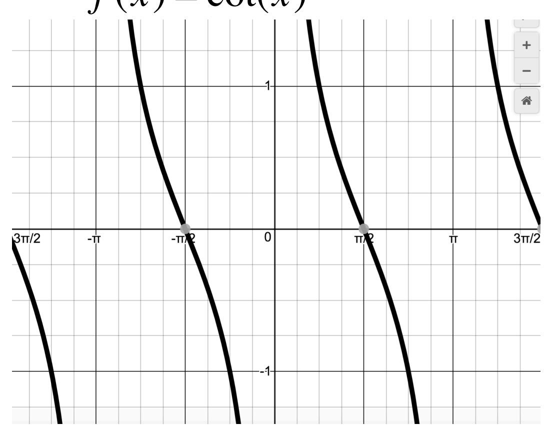


Graph of $f(x) = \cot(x)$

$f(x) = \tan(x)$



$f(x) = \cot(x)$



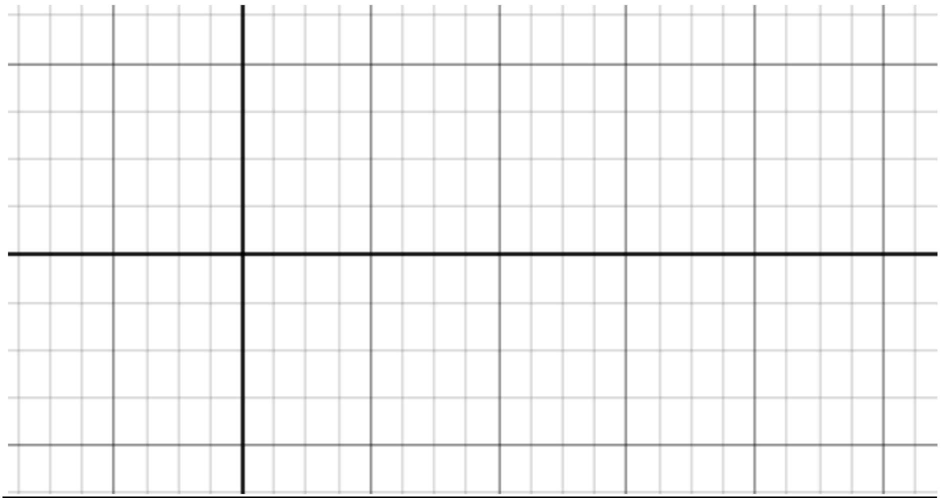
Discuss domain, range, period, odd, asymptotes…

Note: the asymptotes for tangent and cotangent are not in the same location (why?). In addition notice that the tangent graph increases between each pair of asymptotes where the cotangent decreases.
See text for more examples

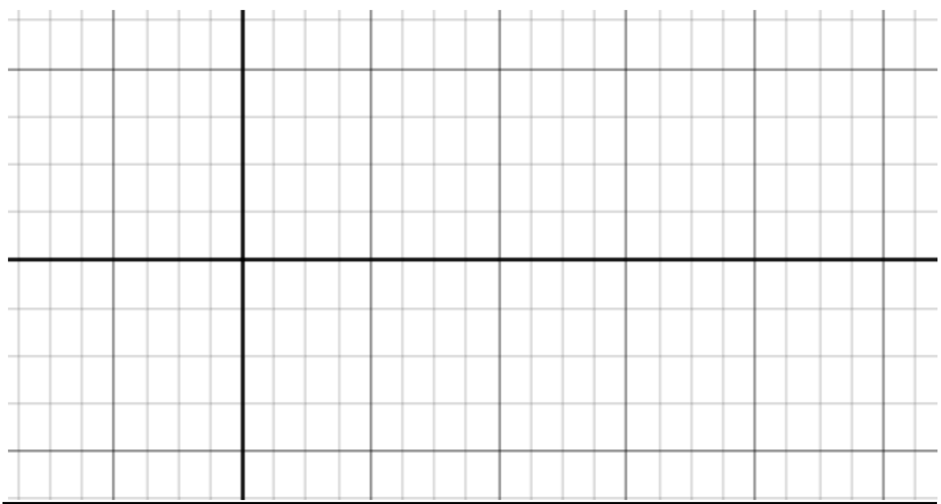
WS: Graphing the other trigonometric functions.

Graph the following. Label asymptotes. Show scale. Check a point.

(1) $f(x)=\tan(\pi x)$



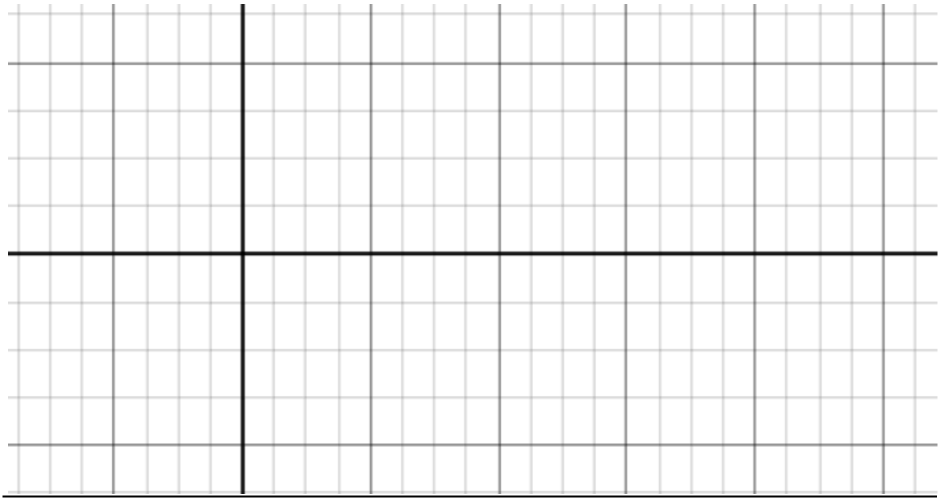
(2) $f(x)=\frac{1}{2}\tan\left(\frac{1}{3}x\right)$



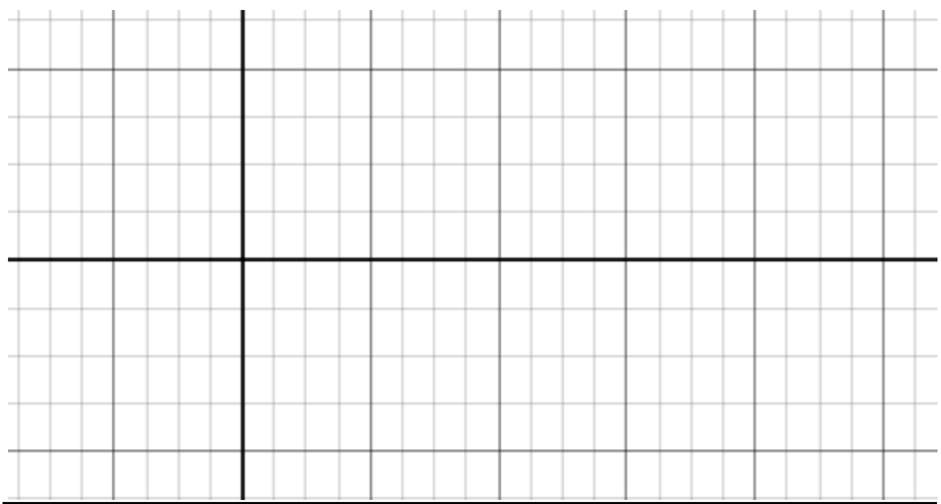
(worksheet cont'd)

(worksheet cont'd)

(3) $f(x) = \sec(5x)$



(4) $f(x) = \csc\left(2x - \frac{\pi}{4}\right)$



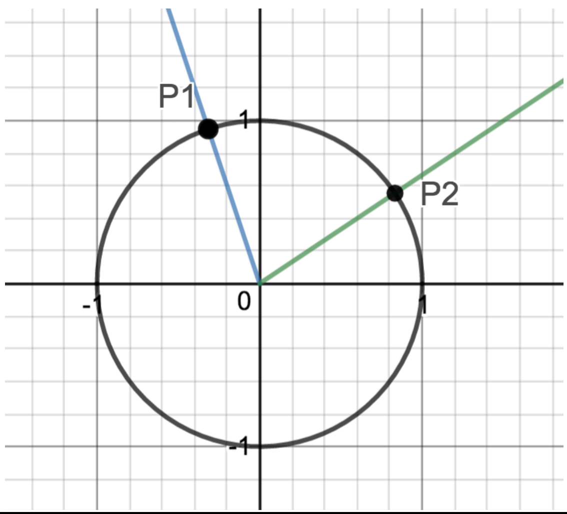
Book does in a little more detail. Practice problems from 10.5 as needed to be able to do problems similar to those on this worksheet. 1, 2, 4-11. No need to turn in.

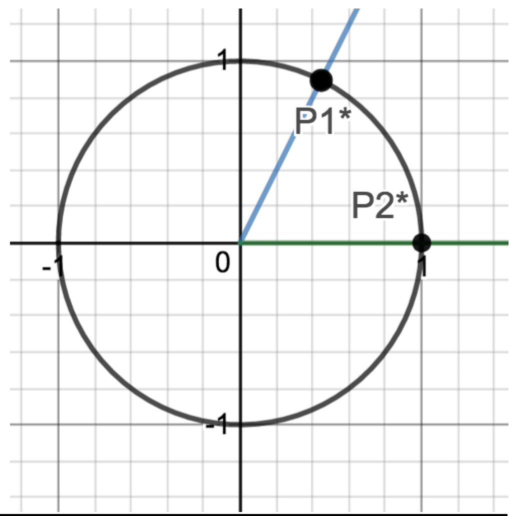
11.2i More Identities

Sum and Difference Formulas:

Recall Function Notation: $\cos\left(\frac{\pi}{2}-\frac{\pi}{3}\right)$ ________________________________

Development of the identity for $\cos(\alpha-\beta)$





Check on example above

We have shown: $\cos(\alpha-\beta)=\cos(\alpha)\cos(\beta)+\sin(\alpha)\sin(\beta)$
which we can use to show:

$$\cos(\alpha+\beta)=$$

Now, making use of the co-function formulas we saw from section B2, (the cosine of an angle equals the sine of its complement),

$$\begin{aligned}
\sin(\alpha+\beta) &= \cos\left(\frac{\pi}{2}-(\alpha+\beta)\right) \\
&= \cos\left(\left[\frac{\pi}{2}-\alpha\right]-\beta\right) \\
&= \cos\left(\frac{\pi}{2}-\alpha\right)\cos(\beta)+\sin\left(\frac{\pi}{2}-\alpha\right)\sin(\beta) \\
&= \sin(\alpha)\cos(\beta)+\cos(\alpha)\sin(\beta)
\end{aligned}$$

From here we can derive

$$\sin(\alpha-\beta)=$$

This leads to:

$$\tan(\alpha-\beta)=$$

And similarly for $\tan(\alpha+\beta)$

$$\sin(a+b)=\sin a\cos b+\cos a\sin b$$
$$\sin(a-b)=\sin a\cos b-\cos a\sin b$$
$$\cos(a+b)=\cos a\cos b-\sin a\sin b$$
$$\cos(a-b)=\cos a\cos b+\sin a\sin b$$
$$\tan(a+b)=\frac{\tan a+\tan b}{1-\tan a\tan b}$$
$$\tan(a-b)=\frac{\tan a-\tan b}{1+\tan a\tan b}$$

Using the sum and difference formulas:

Ex: Find the exact value of $\sin\left(15^\circ\right)$ and $\cos\left(\frac{7\pi}{12}\right)$

Ex: Simplify $\cos(\pi-\theta)$

Ex: Given that $\cos(\alpha)=\frac{5}{13}$, with α in Quadrant I, and $\sin(\beta)=\frac{-1}{4}$ with β in Quadrant III, find $\tan(\alpha-\beta)$.

Ex: Use the fact that $2\theta=\theta+\theta$ to derive a formula for $\sin\left(2\theta\right)$

11.2ii Even More Identities

Double Angle Formulas

Recall Function Notation: $\sin\left(2 \bullet \frac{\pi}{6}\right)$ ______________________________

Using the fact that $2\theta = \theta + \theta$, we can show:

Often we use these as a templates.

Theorem 11.9. Double Angle Identities: For all applicable angles θ,

- $\cos(2\theta) = \begin{cases} \cos^2(\theta) - \sin^2(\theta) \\ 2\cos^2(\theta) - 1 \\ 1 - 2\sin^2(\theta) \end{cases}$
- $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$
- $\tan(2\theta) = \dfrac{2\tan(\theta)}{1 - \tan^2(\theta)}$

Using the double angle formulas:

EX: Given that P(-1,2) lies on the terminal side of θ, find $\cos(2\theta)$.

EX: Find an identity for $\sin(3\theta)$ in terms of θ.

EX: Verify the identity $\cot(2\theta) = \dfrac{\cot^2(\theta) - 1}{2\cot(\theta)}$

More Solving Trigonometric Equations (covered in 11.4 of text, I break into pieces)

Use factoring with zero product law.

(1) Solve: $2\cos^2(\theta)-\cos(\theta)-1=0$ can use substitution

(2) Solve for $0 \le x < 2\pi;$ $\sqrt{3}\sin(x)\tan(x)=\sin(x)$ Note: ________________________

Using Identities to solve equations.

(3) $\cos^2(\theta)-\sin^2(\theta)+\sin(\theta)=0$

(4) $\cos(2\theta)+6\sin^2(\theta)=4$

(5) Solve for $0 \le x < 2\pi;$ $\sin(x)\cos(x)=\frac{1}{4}$

Note: There are many worksheets for practice on the Math 8 page.

WS: Solving Trig Equations part iii

(1) Solve the following equations:

a) $2\cos^2(\theta)-7\cos(\theta)-4=0$ *Ans:* $\theta=\dfrac{2\pi}{3}+2\pi k,\ \dfrac{4\pi}{3}+2\pi k\quad k\in Z$

---

b) $\sin(2x)=\sin(x)$ *Ans:* $x=\dfrac{\pi}{3}+2\pi k,\ \dfrac{5\pi}{3}+2\pi k,\ \pi k,\ k\in Z$

(2) Solve the following equations for $0\le\theta<2\pi$:

a) $\cos(2\theta)+\sin(\theta)=-2$ *Ans:* $\theta=\dfrac{3\pi}{2}$

---

b) $4\sin^2(\theta)-1=0$ *Ans:* $\theta=\dfrac{\pi}{6},\ \dfrac{5\pi}{6},\ \dfrac{7\pi}{6},\ \dfrac{11\pi}{6}$

---

c) $\tan(2\theta)=1$ *Ans:* $\theta=\dfrac{\pi}{8},\ \dfrac{5\pi}{8},\ \dfrac{9\pi}{8},\ \dfrac{13\pi}{8}$

---

d) $3\tan^2(x)-\sec^2(x)-5=0$ *Ans:* $\theta=\dfrac{\pi}{3},\ \dfrac{2\pi}{3},\ \dfrac{4\pi}{3},\ \dfrac{5\pi}{3}$

(3) Solve for $-\dfrac{\pi}{2}\le x\le\dfrac{\pi}{2}$: $4\tan(x)+4=0$ *Ans:* $x=-\dfrac{\pi}{4}$

11.2iii More Identities

Power Reducing and Half Angle Formulas

From the Double Angle Formulas for cosine, we can derive other useful identities.

$\cos(2\theta) = 2\cos^2(\theta) - 1$	$\cos(2\theta) = 1 - 2\sin^2(\theta)$
Power Reducing Formulas	
Half Angle Formulas	

Using the power reducing and half angle formulas:

EX: Write in terms of terms having power of at most 1. (This is a process that will be very useful in calculus)

$$\cos^4(x)$$

EX: Find the exact value of $\cos(112.5^\circ)$ and $\sin(112.5^\circ)$.

Note: When using the half angle formulas you must choose ________________________________ and that choice is based on the quadrant of __________________________

EX: If $\cos(\alpha) = \frac{3}{5};\ \ \frac{3\pi}{2} < \alpha < 2\pi$, find $\cos\left(\frac{\alpha}{2}\right)$ and $\sin\left(\frac{\alpha}{2}\right)$

Product to Sum Formulas *(memorization not required)*

Derivation:

Theorem 11.12. Product to Sum Formulas: For all angles α and β,

- $\cos(\alpha)\cos(\beta) = \frac{1}{2}[\cos(\alpha-\beta)+\cos(\alpha+\beta)]$
- $\sin(\alpha)\sin(\beta) = \frac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)]$
- $\sin(\alpha)\cos(\beta) = \frac{1}{2}[\sin(\alpha-\beta)+\sin(\alpha+\beta)]$

EX: Write as a sum: $\sin(3x)\cos(2x)$

Sum to Product Formulas (also called factoring formulas) *(memorization not required)*

With a substitution of $\alpha-\beta=x$ and $\alpha+\beta=y$, we derive :

Sum to Product Formula

$$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$$

$$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$$

EX: Solve for $0 \le \theta < 2\pi$, $\sin(3\theta)-\sin(\theta)=0$

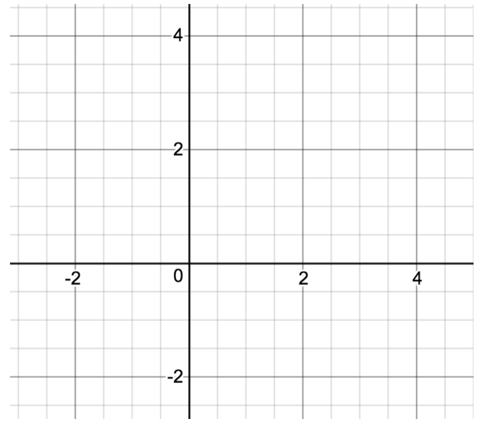
.

11.3i Inverse Trigonometric Functions

<u>Recall Inverse Functions (section 5.6)</u>

Given $f(x) = x^2 - 1; \quad x \geq 0$,

1) Find $f^{-1}(x)$
2) Graph $f(x)$ and $f^{-1}(x)$.
3) Find the domain and range of $f(x)$ and $f^{-1}(x)$.
4) Find $(f \circ f^{-1})(x)$ and $(f^{-1} \circ f)(x)$

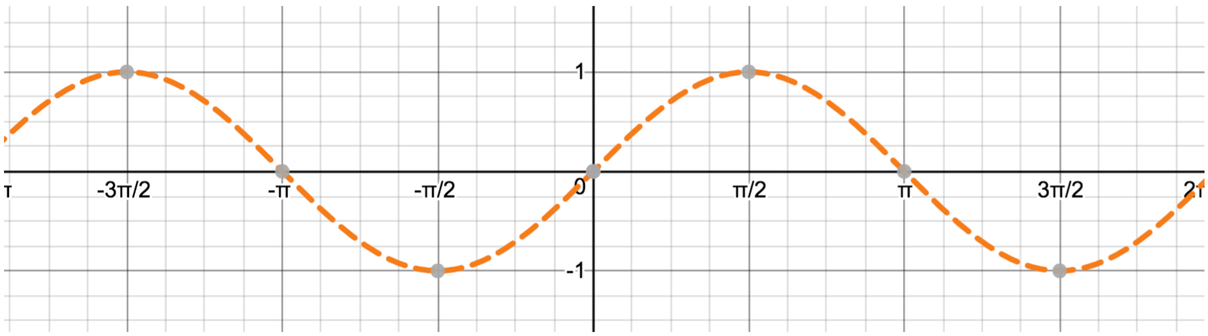


Why is the restriction $x \geq 0$ necessary?____________________________

$$f^{-1}(f(x)) = x \quad \text{where } x \text{ is in the domain of } f$$
$$f(f^{-1}(x)) = x \quad \text{where } x \text{ is in the domain of } f^{-1}$$

Inverse Sine Function

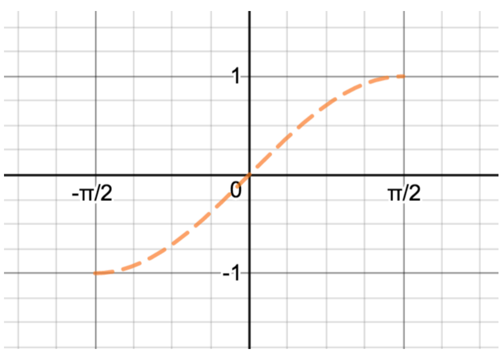
Does $g(x) = \sin(x)$ have an inverse? ____



What restriction would we need to make so that at least a piece of this function has an inverse?

Given $f(x) = \sin(x)$; ___________________

1) Find $f^{-1}(x)$
2) Graph $f(x)$ and $f^{-1}(x)$.
3) Find the domain and range of $f(x)$ and $f^{-1}(x)$.



We define $y = \sin^{-1}(x)$ or $y = \arcsin(x)$ to mean $\begin{cases} \sin(y) = x \\ \quad AND \\ -\frac{\pi}{2} \le y \le \frac{\pi}{2} \end{cases}$

Note: Both the input and output of this function are real numbers, but it is sometimes helpful to think in terms of angles.

that is let $\theta = \sin^{-1}(x)$ or $\theta = \arcsin(x)$ mean $\begin{cases} \sin(\theta) = x \\ \quad AND \\ -\frac{\pi}{2} \le \theta \le \frac{\pi}{2} \end{cases}$

For example:

$\sin\left(\frac{\pi}{6}\right)=\frac{1}{2}$ $\qquad$ $\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}$

$\sin(angle)=number$	$\sin^{-1}(number)=angle\ in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

<u>Finding exact values of the inverse sine function for special inputs: (like:______________________)</u>

Ex: $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Set $\theta=\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ and re-write according to the definition as ___________________

In words: $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ *is the real number (or angle) in* $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ *whose sine (or y value on the unit circle) is* $\frac{\sqrt{3}}{2}$

Ex: $\sin^{-1}\left(\frac{-\sqrt{2}}{2}\right)$

Since $y=\sin^{-1}(x)$ is a *function*, _________________________________

Ex:

Ex: $\sin^{-1}(2)$______________________

Ex: $\sin^{-1}\left(\frac{1}{4}\right)$ ______________________

<u>Compositions</u>

$\sin^{-1}\left(\sin\left(\frac{\pi}{3}\right)\right)=$ ______________ $\qquad$ $\sin\left(\sin^{-1}(1)\right)=$___________________

$\sin^{-1}\left(\sin\left(\frac{5\pi}{6}\right)\right)=$______________ $\qquad$ $\sin\left(\sin^{-1}(2)\right)=$___________________

$$f^{-1}(f(x)) = \sin^{-1}(\sin x) = x \qquad \text{where } -\frac{\pi}{2} \le x \le \frac{\pi}{2}$$

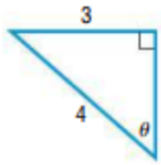
$$f(f^{-1}(x)) = \sin(\sin^{-1} x) = x \qquad \text{where } -1 \le x \le 1$$

Think about it: $\sin^{-1}\left(\sin\left(\frac{4\pi}{5}\right)\right)$=________________

Using the inverse sine function

Right triangle applications:

1) Find θ in the given triangle.



2)

A parasailor is being pulled by a boat on Lake Ippizuti. The cable is 300 feet long and the parasailor is 100 feet above the surface of the water. What is the angle of elevation from the boat to the parasailor? Express your answer using degree measure rounded to one decimal place.

Solving Equations:
We did this same process previously, for where inputs where key number/angle.

Examples: While you are learning the process, I highly encourage you to draw the unit circle and find the location of the terminal sides corresponding to the solution.

Review: Solve: $\sin(t) = \frac{1}{2}$ for $0 \le t < 2\pi$

This is asking us to find the real number (arc length or corresponding angle, in radians) whose corresponding point on the unit circle has____________value of ½. This occurs at one of our key number/angle inputs. They are "π/6 type" inputs, that is they have a ________________ of π/6 .

Solutions: ______________________

Now Solve: $\sin(t) = \frac{1}{3}$ for $0 \le t < 2\pi$

This is asking us to find the real number (arc length or corresponding angle, in radians) whose corresponding point on the unit circle has____________value of 1/3. This is not one of our known inputs.

What would the reference angle be? ______________

Solutions: ______________________

Review: Solve: $\sin(t) = \frac{-\sqrt{3}}{2}$ for $0 \le t < 2\pi$

This is asking us to find the real number (arc length or corresponding angle, in radians) whose corresponding point on the unit circle has_____________value of $-\frac{\sqrt{3}}{2}$. This occurs at one of our key number/angle inputs whose reference angle is ___________________.

Solutions: ________________________

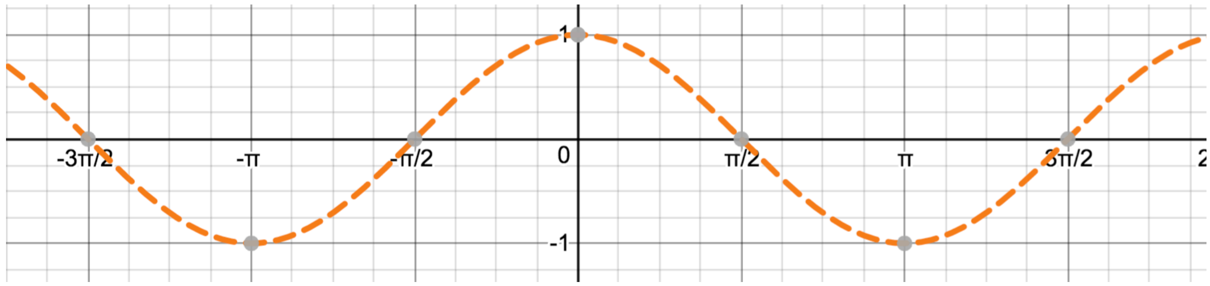
Now Solve: $\sin(t) = \frac{-3}{4}$ for $0 \le t < 2\pi$

This is asking us to find the real number (arc length or corresponding angle, in radians) whose corresponding point on the unit circle has_____________value of -3/4. This is not one of our known inputs.

What would the reference angle be? ________________

Solutions: ________________________

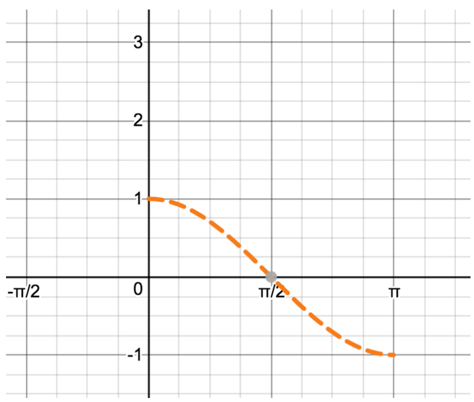
Inverse Cosine Function



What restriction would we need to make so that at least a piece of this function has an inverse?

Given $f(x) = \cos(x)$; ____________________

1) Find $f^{-1}(x)$
2) Graph $f(x)$ and $f^{-1}(x)$.
3) Find the domain and range of $f(x)$ and $f^{-1}(x)$.



We define $y = \cos^{-1}(x)$ or $y = \arccos(x)$ to mean $\begin{cases} \cos(y) = x \\ \quad AND \\ 0 \le y \le \pi \end{cases}$

Note: Both the input and output of this function are real numbers, but it is sometimes helpful to think in terms of angles.

that is let $\theta = \cos^{-1}(x)$ or $\theta = \arccos(x)$ mean $\begin{cases} \cos(\theta) = x \\ \quad AND \\ 0 \le \theta \le \pi \end{cases}$

As before, it is helpful to think of the input/outputs as follows:

$\cos(angle) = number$ $\qquad\qquad$ $\cos^{-1}(number) = angle\ in\ [0,\ \pi]$

Finding exact values of the inverse sine function for special inputs:

Ex: $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$

In words: $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$ *is the real number (or angle) in* $[0, \pi]$ *whose cosine (or x value on the unit circle) is* $\frac{\sqrt{2}}{2}$

Ex: $\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$

Ex:

Ex: $\cos^{-1}(-0.2)$ ______________________

Compositions

$\cos^{-1}\left(\cos\left(\frac{\pi}{6}\right)\right)$= _____________ $\cos\left(\cos^{-1}(0.5)\right)$=__________________

$\cos^{-1}\left(\cos\left(\frac{5\pi}{3}\right)\right)$=_____________ $\cos\left(\cos^{-1}(1.5)\right)$=__________________

$$f^{-1}(f(x)) = \cos^{-1}(\cos x) = x \qquad \text{where } 0 \le x \le \pi$$
$$f(f^{-1}(x)) = \cos(\cos^{-1} x) = x \qquad \text{where } -1 \le x \le 1$$

Think about it: $\cos^{-1}\left(\cos\left(\frac{13\pi}{12}\right)\right)$=__________________

Solving equations using inverse cosine:

Review: Solve: $\cos(t) = \frac{\sqrt{3}}{2}$ for $0 \le t < 2\pi$

This is asking us to find the real number (arc length or corresponding angle, in radians) whose corresponding point on the unit circle has_____________value of $\frac{\sqrt{3}}{2}$. This occurs at one of our key number/angle inputs. They are "π/6 type" inputs, that is they have a __________________ of π/6 .

Solutions: _________________________

Now Solve: $\cos(t) = \frac{1}{3}$ for $0 \le t < 2\pi$

This is asking us to find the real number (arc length or corresponding angle, in radians) whose corresponding point on the unit circle has_____________value of 1/3. This is not one of our known inputs.

What would the reference angle be? ________________

Solutions? ____________________________

Review: Solve: $\cos(t) = \frac{-\sqrt{2}}{2}$ for $0 \le t < 2\pi$

This is asking us to find the real number (arc length or corresponding angle, in radians) whose corresponding point on the unit circle has_____________value of $-\frac{\sqrt{2}}{2}$. This occurs at one of our key number/angle inputs whose reference angle is ____________________.

Solutions: _________________________

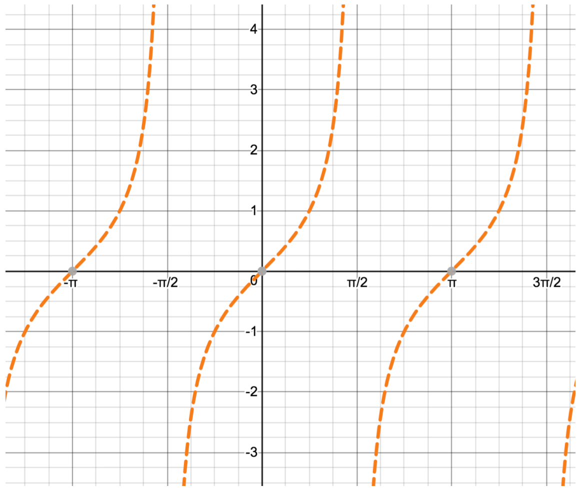
Now Solve: $\cos(t) = \frac{-1}{4}$ for $0 \le t < 2\pi$

This is asking us to find the real number (arc length or corresponding angle, in radians) whose corresponding point on the unit circle has_____________value of -1/4. This is not one of our known inputs.

What would the reference angle be? ________________

Solutions: _________________________

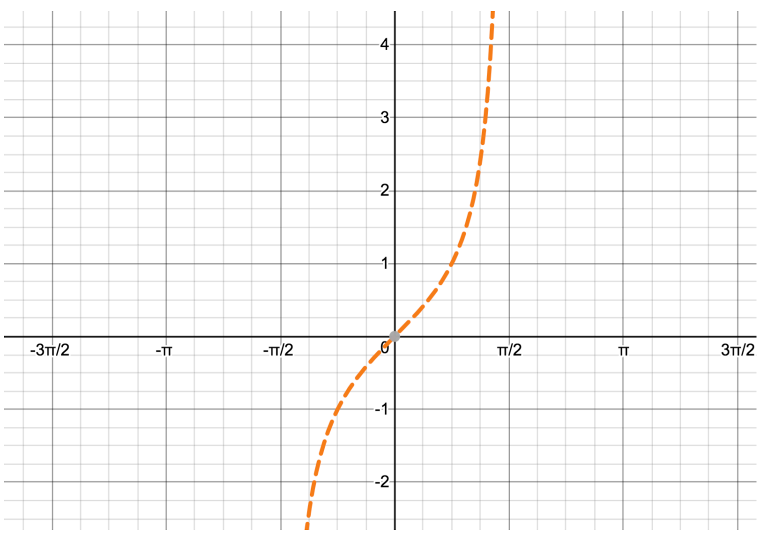
Inverse Tangent Function



What restriction would we need to make so that at least a piece of this function has an inverse?

Given $f(x) = \tan(x)$; ____________________

1) Find $f^{-1}(x)$
2) Graph $f(x)$ and $f^{-1}(x)$.
3) Find the domain and range of $f(x)$ and $f^{-1}(x)$.



We define $y = \tan^{-1}(x)$ or $y = \arctan(x)$ to mean $\begin{cases} \tan(y) = x \\ \quad AND \\ -\dfrac{\pi}{2} < y < \dfrac{\pi}{2} \end{cases}$

As before, both the input and output of this function are real numbers, but it is sometimes helpful to think in terms of angles.

$\tan(angle) = number$ $\tan^{-1}(number) = angle\ in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Finding exact values of the inverse sine function for special inputs:

Ex: $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$

In words: $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$ *is the real number (or angle) in* $[0, \pi]$ *whose tangent* $\frac{\sqrt{3}}{3}$

Ex: $\tan^{-1}(-1)$

Ex:

Ex: $\tan^{-1}(8)$ ______________________

Compositions

Similar to the case for cosine and sine,

$$f^{-1}(f(x)) = \tan^{-1}(\tan x) = x \quad \text{where } -\frac{\pi}{2} < x < \frac{\pi}{2}$$
$$f(f^{-1}(x)) = \tan(\tan^{-1} x) = x \quad \text{where } -\infty < x < \infty$$

Solving equations using inverse tangent:

Solve: $\tan(t) = 4$ for $0 \le t < 2\pi$

What would the reference angle be? ________________

Solutions? ___________________________

Solve: $\tan(t) = -\frac{3}{5}$ for $0 \le t < 2\pi$

What would the reference angle be? ________________

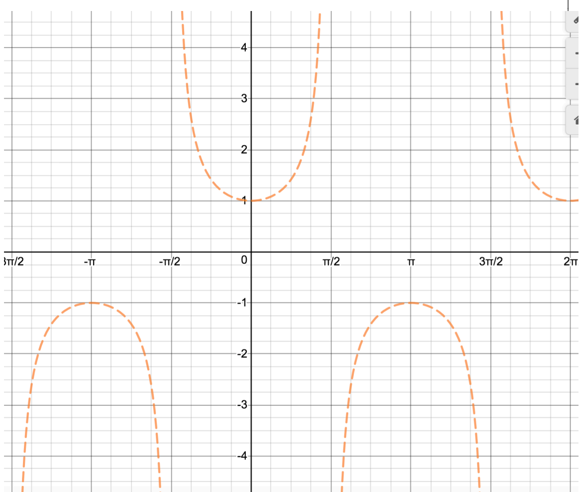
Solutions? ___________________________

What if we were asked for ALL solutions? _____________________________-

11.3ii Inverse Trigonometric Functions – The other inverse functions and mixed compositions

The other inverses:

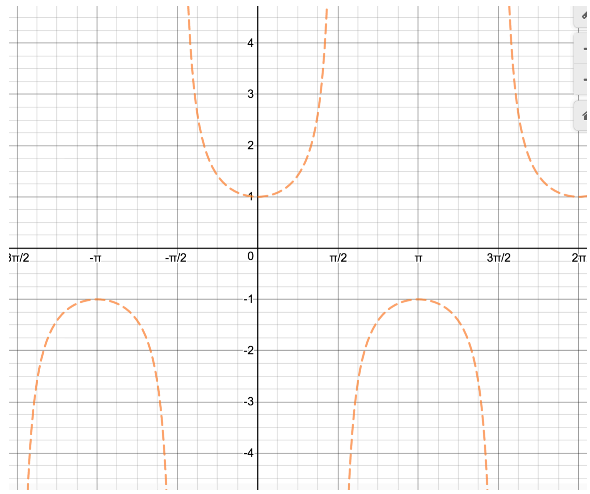
$f(x) = \sec(x)$



“Trig friendly” restrictions: ______________

$\sec^{-1}(2) =$ _______________

$\sec^{-1}(-\sqrt{2}) =$ _______________



“Calculus friendly” restrictions:________________

$\sec^{-1}(2) =$ _______________

$\sec^{-1}(-\sqrt{2}) =$ _______________

See the book for $\csc^{-1}(x)$ and $\cot^{-1}(x)$. You do not need to memorize these restrictions, but know how to find values for a given set of restrictions.

<u>Mixed Compositions – common in calculus</u>

Find exact values:

$\sin\left(\cos^{-1}\left(\frac{3}{5}\right)\right)$

$\tan\left(\sin^{-1}\left(\frac{-2\sqrt{5}}{5}\right)\right)$

$\sin(2\arctan(3))$

$\cos\left(\tan^{-1}\left(\frac{1}{4}\right)-\sin^{-1}\left(\frac{-2}{3}\right)\right)$